

THE EFFICIENCY OF ELECTRON ACCELERATION IN COLLISIONLESS SHOCKS AND GAMMA-RAY BURST ENERGETICS

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ABSTRACT

Afterglow observations are commonly used to determine the parameters of GRB explosions, the energy E , surrounding density n , postshock magnetic field equipartition fraction ϵ_B , and electron equipartition fraction ϵ_e , under the frequently made assumption that the efficiency of electron “injection” into relativistic shock acceleration is high, i.e., that the fraction f of electrons that undergo acceleration is $f \approx 1$. We show that the value of f cannot be determined by current observations, since currently testable model predictions for a parameter choice $\{E' = E/f, n' = n/f, \epsilon'_B = f\epsilon_B, \epsilon'_e = f\epsilon_e\}$ are independent of the value of f for $m_e/m_p \leq f \leq 1$. Current observations imply that the efficiency f is similar for highly relativistic and subrelativistic shocks and plausibly suggest that $f \sim 1$, quite unlike the situation in the Crab Nebula. However, values $m_e/m_p \leq f \ll 1$ cannot be ruled out, implying a factor m_e/m_p uncertainty in determination of model parameters. We show that early, ≤ 10 hr, radio afterglow observations, which will be far more accessible in the *Swift* era, may provide constraints on f . Such observations will therefore provide a powerful diagnostic of GRB explosions and of the physics of particle acceleration in collisionless shocks.

Subject headings: acceleration of particles — gamma rays: bursts — gamma rays: theory — radiation mechanisms: nonthermal — shock waves

1. INTRODUCTION

Synchrotron emission by shock-accelerated particles is central to our understanding of explosive, high-energy astrophysical phenomena, such as supernova remnants, jets from active galactic nuclei (AGNs) and quasars, plerionic nebulae, and γ -ray burst (GRB) afterglows. GRB afterglows have provided an unprecedented opportunity for diagnosing the blast wave and attendant shock acceleration, because their brevity in the observer’s time frame and ultrahigh Lorentz factors allow rapid evolution of the synchrotron spectrum, which can be observed over a wide span of wavelength regimes in real time.

The afterglow radiation of GRBs (e.g., Kulkarni et al. 2000) can be naturally explained as due to synchrotron emission of electrons accelerated in relativistic collisionless shocks driven by the GRB explosion into the medium surrounding the GRB progenitor (for reviews, see Piran 2000; Mészáros 2002; Waxman 2003). The energy released in the explosion leads to the formation of a diverging shock wave, which propagates into the ambient plasma. At a sufficiently late time (at times much longer than the burst duration) all the explosion energy is carried by the shocked ambient plasma (a vanishingly small fraction of the energy remains in the ejecta produced by the explosion). The radiation is believed to be produced by electrons of the ambient medium, which are accelerated to high energy as they pass through the diverging shock.

The dynamics of a spherical shock wave is determined by the explosion energy E and by the surrounding medium number density n . If the initial GRB outflow is jetlike, an additional parameter, the jet opening angle θ_j , is required in order to specify the flow. For a given shock dynamics, the luminosity and

spectrum of emitted radiation are then determined by the fractions ϵ_B and ϵ_e of shock thermal energy carried, respectively, by magnetic field and electrons, and by the shape of the electron distribution function. The fraction of explosion energy E converted to thermal energy in the shock is determined by the hydrodynamics and is of order unity. The electron and magnetic field energy densities are therefore proportional to $\epsilon_e E$ and $\epsilon_B E$, respectively. The electron distribution function is commonly assumed to be a power law of index $p \equiv -d \ln n_e / d \ln \epsilon_e$, where ϵ_e is the electron energy, above some minimum energy ϵ_{e0} (we use ϵ to denote single-particle energy and E to denote the total flow energy).

The processes of magnetic field generation and electron injection in collisionless shocks are not understood from basic principles, and ϵ_B , ϵ_e , and ϵ_{e0} cannot at present be determined theoretically. Rather, they are treated as free parameters of the model, constrained by observations. It is important to note here that ϵ_{e0} is, in general, an independent parameter of the model. It is a function not only of ϵ_e but also of the fraction f of electrons assumed to be accelerated to beyond ϵ_{e0} . It is commonly assumed, however, that $f = 1$, in which case ϵ_{e0} is uniquely determined by the other model parameters.

The observed afterglow synchrotron spectra constrain p , as is well known, and the break in the power-law decay of afterglow flux is widely believed (Rhoads 1999) to establish the opening angle θ_j in terms of E/n . Under the assumption that *all* the ambient electrons were injected to beyond ϵ_{e0} , i.e., $f = 1$, the remaining four parameters $\{\epsilon_B, \epsilon_e, n, E\}$ are fixed by the four observables, ν_m (the frequency of maximum intensity), F_m (the intensity at ν_m), ν_{cool} (the synchrotron cooling break), and ν_a (the self-absorption frequency).

The results that have been deduced for p , ϵ_B , and ϵ_e are consistent with both current knowledge and current ignorance about shock acceleration: in bursts where p can be determined accurately (e.g., Waxman 1997a; Galama et al. 1998a; Frail et al. 2000; Stanek et al. 1999) $p = 2.2 \pm 0.1$ is inferred. This value

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is consistent with the theoretical value of p derived for test-particle acceleration in relativistic shocks via the first-order Fermi mechanism, assuming isotropic diffusion of particles in momentum space, $p = 2.22 \pm 0.02$ obtained in numerical calculations (Bednarz & Ostrowski 1998; Kirk et al. 2000; Achterberg et al. 2001), and $p = 20/9$ obtained by a more recent analytic analysis (Keshet & Waxman 2005). This value of p is not consistent with test-particle results for large-angle scattering in relativistic shocks, which produce very hard spectra. It is, however, consistent with the value expected in the 100 MeV–10 GeV range by nonlinear theory for cosmic-ray-mediated shock (Ellison & Eichler 1985; Ellison & Double 2002). Despite the agreement of the observed and theoretically derived values of p , assuming isotropic diffusion, it should be kept in mind that questions remain about diffusive shock acceleration, particularly with regard to relativistic generalization and electron injection, and that there are alternative acceleration processes (e.g., Arons & Tavani 1994; Nishikawa et al. 2005; Hededal et al. 2004).

It is natural to hope that the values of ϵ_B and ϵ_e are universal, since they are determined by the microphysics of the collisionless shock. The constancy of p and of ϵ_e among different bursts is strongly supported by observations. Universal values of p and ϵ_e , $p \approx 2$ and $\epsilon_e \approx 0.1$, typically inferred from most optical afterglows, are also inferred from the clustering of explosion energies (Frail et al. 2001) and from X-ray afterglow luminosity (Freedman & Waxman 2001; Berger et al. 2003).³ The value of ϵ_B is less well constrained by observations. However, in cases where ϵ_B can be reliably constrained by multi-waveband spectra, values close to equipartition are inferred (e.g., Frail et al. 2000). Such high values for ϵ_B and ϵ_e are remarkable and beg for an explanation. The magnetic field required for allowing electron acceleration and emission of synchrotron radiation may conceivably be produced in the collisionless shock driven by the GRB explosion by Weibel instabilities or the like (see, e.g., Blandford & Eichler 1987; Gruzinov & Waxman 1999; Medvedev & Loeb 1999), or it may be that the accelerated particles mix with the magnetic field of the fireball itself.

No less surprising is the conclusion by Waxman (1997b) that ϵ_{e0} is close to $\gamma m_p c^2$ and that the low-frequency radio spectra imply that there are relatively few electrons in the decade or two just below ϵ_{e0} . Had the electrons been picked up by shock acceleration at some energy much lower than $\gamma m_p c^2$, the power-law spectrum imparted by the shock acceleration would have extended down to much lower energies, and only a small minority of them would have made it to $\gamma m_p c^2$ or higher. In the case of the Crab Nebula, for example, which contains perhaps the best-studied relativistic shock wave, this is indeed the case: most of the electrons in the nebula emit in the radio and probably have Lorentz factors of order 10^2 , which is many orders of magnitude lower than $\gamma m_p c^2$ and even about a factor of 10^2 below $\gamma m_e c^2$. More is said about this below. While this paper does not aim to explain this gaping difference between afterglows and the Crab Nebula, it motivates us to check the assumption that $f = 1$ in the case of the former.

In any case, we are unable to determine from basic principles the efficiency of electron “injection” to beyond some threshold energy well beyond $\gamma m_e c^2$. Even when the number of electrons beyond some injection threshold ϵ_{e0} is known, we are unable to

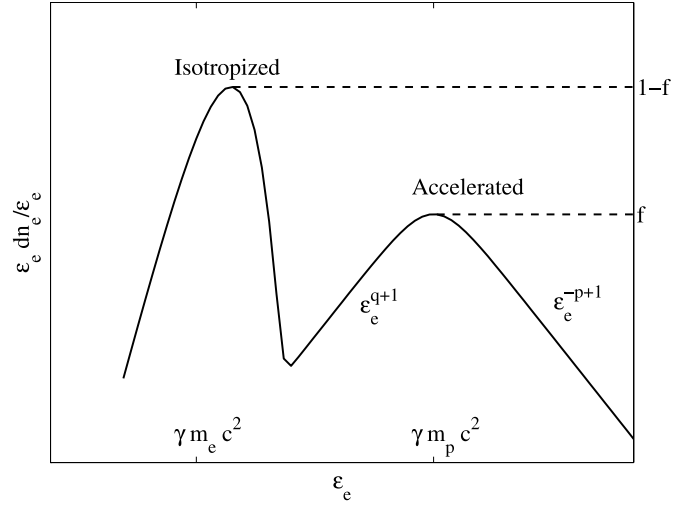


FIG. 1.—Schematic representation of the postshock electron distribution, for a relativistic shock of Lorentz factor γ (or subrelativistic shock of velocity v). Scattering of electrons streaming toward the shock with Lorentz factor γ (or velocity v) results in postshock “thermal” energy of $\sim \gamma m_e c^2$ (or $\sim m_e v^2$). A fraction f of the electrons is assumed to be injected into the acceleration process, which significantly increases the average energy of these electrons, to $\sim \epsilon_{e0}$, and produces a power-law distribution at $\epsilon_e > \epsilon_{e0}$. As we show here, afterglow observations imply $\epsilon_{e0} \sim \gamma m_p c^2$ in the relativistic phase and $\epsilon_{e0} \sim m_p v^2$ in the subrelativistic phase but do not allow us to determine f . Afterglow observations also require an electron number density that increases with energy sufficiently fast, $q \equiv d \ln n_e / d \ln \epsilon_e > 0$, over ~ 1.5 decades of energy below ϵ_{e0} (Waxman 1997b).

determine theoretically the fraction f of total electrons that these high-energy electrons represent. It is conceivable that a large fraction, $1 - f \sim 1$, of the electron population does not participate in the acceleration process and remains well below ϵ_{e0} . This is discussed in § 2. In § 3 we discuss observational signatures of the existence of such *noninjected* thermal electrons in GRB-induced blast waves. Our main results and their implications are summarized in § 4. We discuss both the implications to GRB phenomenology and the implications for the theory of collisionless shock acceleration, in particular in the context of constraints imposed by observations on astrophysical systems other than GRBs.

2. MODEL PREDICTION DEGENERACY

To clarify the issues involved in the electron injection problem let us consider the situation illustrated in Figure 1, which may arise for a relativistic shock propagating with Lorentz factor $\gamma \gg 1$ (or subrelativistic shock propagating with velocity $v \ll c$) into a cold plasma of protons and electrons (as may be the case for a shock driven by a GRB explosion into the interstellar medium [ISM]). In the shock frame, a cold stream of protons and electrons approaches the shock with Lorentz factor γ (velocity v). The particles are being scattered at the shock front, resulting in a velocity distribution that is close to isotropic behind the shock, thus converting a large fraction of the kinetic energy of the incoming flow to thermal energy. Isotropization of the electron and proton incoming flow would lead to a postshock proton “temperature” $T_p \sim \gamma m_p c^2$ (or $T_p \sim m_p v^2$) and to a postshock electron “temperature” $T_e \sim \gamma m_e c^2 \ll T_p$ (or $T_e \sim m_e v^2$). In order for the electrons to gain a significant fraction of the postshock thermal energy, some process must couple them to the protons and accelerate them to energy $\gg T_e$. This process is yet unknown, and we cannot determine based on theoretical considerations what fraction of the electrons are being accelerated. Thus, in addition

³ Apparently deviant values of p (Chevalier & Li 1999; Panaitescu & Kumar 2002) are inferred based on light curves, rather than spectra, and are sensitive to model assumptions (e.g., they depend on the assumed radial dependence of the ambient medium density).

to ϵ_e , the acceleration process must be described by (at least) one additional parameter, the fraction f of accelerated electrons. We show here that afterglow observations imply $\epsilon_{e0} \sim \gamma m_p c^2$ in the relativistic phase and $\epsilon_{e0} \sim m_p v^2$ in the subrelativistic phase but do not allow one to determine f .

As pointed out in Waxman (1997b), the energy distribution of electrons below the characteristic acceleration energy, ϵ_{e0} , is constrained by radio observations. The slope of the radio afterglow spectrum observed in several cases (e.g., Fig. 1 of Galama et al. 1998a), $f_\nu \propto \nu^{1/3}$, is consistent with that expected for radiation emitted by electrons at ϵ_{e0} at frequencies well below their characteristic synchrotron frequency, which is somewhat below optical at the observation time (typically of order days). In order for the emission from lower-energy electrons, at $\epsilon_e < \epsilon_{e0}$, not to modify this spectrum, $q \equiv d \ln n_e / d \ln \epsilon_e > -1/3$ is required. A somewhat more stringent constraint may be obtained from the requirement that the self-absorption optical depth produced by these electrons not be large enough to affect the observed self-absorption frequency ν_a , $q \equiv d \ln n_e / d \ln \epsilon_e > 2/3$ (provided of course that ν_a is unambiguously established by electrons at ϵ_{e0}). The uncertainty in the value of ν_a (and the values of other characteristic frequencies) determined by observations relaxes the latter constraint to $q \equiv d \ln n_e / d \ln \epsilon_e \gtrsim 0$ (Waxman 1997b). These statements hold for about 1.5 decades of energy below ϵ_{e0} , i.e., the energy range over which the electron distribution would affect the radio emission that has been observed to date. Determining the electron spectrum below ϵ_{e0} is of interest for both GRB phenomenology and particle acceleration theory, suggesting that more careful analysis of radio spectra is warranted for obtaining better constraints on q . This is, however, beyond the scope of the current paper, in which we focus on the injection efficiency f .

In what follows we discuss the degeneracy of afterglow model predictions, showing that the predictions obtained assuming $f = 1$ for some choice of model parameters $\{E, n, \epsilon_B, \epsilon_e\}$ are the same as those obtained for any value of f , $m_e/m_p \leq f \leq 1$, and $\{E' = E/f, n' = n/f, \epsilon'_B = f\epsilon_B, \epsilon'_e = f\epsilon_e\}$. In § 2.1 we discuss the hydrodynamics of the flow, and in § 2.2 we discuss emission of radiation.

2.1. Hydrodynamics

The apparent physical size of the radiation-emitting region has been determined in several cases. During the relativistic stage of shock expansion, the size of the emitting region has been determined directly through the observation of the suppression of diffractive radio scintillation (Goodman 1997; Waxman et al. 1998) and through very long baseline radio interferometry (Taylor et al. 2004). At the subrelativistic stage, the size was determined indirectly through modeling the radio spectrum (Frail et al. 2000; Berger et al. 2004). These observations were used to determine the values of model parameters that determine the flow pattern, E and n , under the assumption of high efficiency, $f = 1$. Modification, due to changes in the value of f , of the values of E and n are therefore allowed, provided these changes do not modify the flow pattern.

We demonstrate here that the velocity field $\mathbf{v}(\mathbf{r}, t)$ associated with the afterglow stage of the GRB explosion depends on the explosion energy E and surrounding medium density n only through the ratio E/n . If the surrounding medium density is not uniform, then for a given functional dependence $g(\mathbf{r})$ of the density on coordinates, $n(\mathbf{r}) = n_0 g(\mathbf{r})$, where n_0 is some normalization, then the velocity field $\mathbf{v}(\mathbf{r}, t)$ depends on E and n_0 only through the ratio E/n_0 .

It is straightforward to demonstrate the validity of the above statements through examination of the hydrodynamic equations. These can be written as

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu (n u^\mu) = 0, \quad (1)$$

where u^μ is the four-velocity tensor and $T^{\mu\nu}$ is the energy-momentum tensor. For the GRB afterglow flows, it is appropriate to assume an ideal fluid flow, i.e., $T^{\mu\nu} = \eta^{\mu\nu} p + (u^\mu u^\nu / c^2)(p + e)$, where p and e are the fluid pressure and (proper) energy density, respectively, and an ideal gas equation of state, $e = n m_p c^2 + (\hat{\gamma} - 1)p$, where $\hat{\gamma}$ is the adiabatic index ($\hat{\gamma} = 4/3$ and $5/3$ for relativistic and nonrelativistic particles, respectively). It is now evident that if $\{u^\mu(\mathbf{r}, t), n(\mathbf{r}, t), p(\mathbf{r}, t)\}$ is a solution of the flow equations, then multiplying the density and pressure by a constant K and leaving the velocity field unchanged provides another solution of the equations, $\{u^\mu(\mathbf{r}, t), n' = K n(\mathbf{r}, t), p' = K p(\mathbf{r}, t)\}$. Since the energy-momentum tensor of the new solution $T'^{\mu\nu}$ is related to the energy-momentum tensor of the original solution $T^{\mu\nu}$ by $T'^{\mu\nu} = K T^{\mu\nu}$, the energy of the flow in the modified solution is larger by a factor K compared to the energy of the flow in the original solution.

The argument given in the previous paragraph proves the statement that $\mathbf{v}(\mathbf{r}, t)$ associated with the afterglow stage of the GRB explosion depends on the explosion energy E and surrounding medium density $n(n_0)$ only through the ratio E/n (E/n_0). Since this argument is, however, rather abstract, it may be useful to examine in some detail how the afterglow flow is affected at various stages as E and n are changed. This examination is also useful for the discussion in § 2.2. For simplicity, we assume in the following discussion a uniform density of the surrounding medium. (It is straightforward to generalize the discussion to a nonuniform density.)

Let us first consider the flow associated with a spherical, relativistic blast wave. When the shock radius R is sufficiently large, compared to the size of the region of initial energy deposition, the flow becomes self-similar, with shock Lorentz factor determined by energy conservation, $E \propto \gamma^2 n R^3$ implying $\gamma \propto (E/n)^{1/2} R^{-3/2}$ (Blandford & McKee 1976). The self-similar flow is therefore completely determined by the ratio E/n .

The early afterglow, on minute timescale, is produced at the onset of the interaction of relativistic GRB plasma with the surrounding medium. At this stage, the highly relativistic plasma ejected by the GRB engine with Lorentz factor γ_i , the “fireball,” drives a forward shock into the surrounding medium, and a reverse shock is driven back into the fireball and decelerates it. Once the reverse shock crosses the fireball plasma shell, the flow approaches the self-similar behavior described above. This transition stage takes place (e.g., Waxman 2003) at a radius that is the larger of (1) the radius at which the self-similar Lorentz factor $\gamma \propto (E/n)^{1/2} R^{-3/2}$ drops below γ_i and (2) the radius at which the thickness of the shocked plasma shell in the self-similar solution, $R/\gamma^2 \propto (n/E) R^4$, exceeds the thickness Δ of the plasma shell ejected by the GRB. The transition radius depends, therefore, on E and n only through the ratio E/n .

If the fireball is jetlike rather than spherical, then flow is well described as a conical section of a spherical fireball as long as the jet opening angle is $\theta_j > 1/\gamma$. In this case, E should be understood as the “isotropic equivalent energy,” the energy that would have been carried by the blast wave had it been spherically symmetric. When the fireball decelerates to $\gamma < 1/\theta_j$, the jet is frequently assumed to expand sideways (Rhoads 1999).

The condition $\gamma \sim 1/\theta_j$ implies that the radius R_j at which sideways expansion begins is given by $(n/E)R_j^3 \propto \theta_j^2$. Therefore, R_j depends on E and n only through the ratio E/n .

During the stage of sideways expansion, the jet does not significantly propagate radially. Finally, after the stage of sideways expansion, the flow becomes subrelativistic (Frail et al. 2000; Livio & Waxman 2000) and, if it becomes a spherical blast wave, the renewed expansion is described by the Sedov–von Neumann–Taylor solutions. At this stage the time dependence of the shock radius is $R \propto (E\theta_j^2/n)^{1/5} t^{2/5}$ (see, e.g., Zel'dovich & Raizer 2002, chap. 12). Here t is the time, and the true energy of the explosion, corrected for the jetlike geometry, is $E_T = E\theta_j^2/2$. Thus, at this stage as well the flow depends on E and n only through the ratio E/n .

2.2. Radiation

Afterglow observations at all stages of flow evolution from the non–self-similar onset of fireball interaction with surrounding gas through the self-similar expansion phase and subsequent jet expansion phase (if present), and including the final subrelativistic phase, are consistent with synchrotron emission of radiation from electrons accelerated to a distribution of the type shown in Figure 1 with high efficiency, $f \approx 1$. Under the assumption $f = 1$, the values of model parameters $\{E, n_0, \epsilon_B, \text{ and } \epsilon_e\}$ (as well as θ_j and p) are determined. Let us now consider what modifications are introduced by allowing $f \ll 1$. We argue that the emission of radiation from shock-accelerated electrons from a flow with parameter choice $\{E' = E/f, n'_0 = n_0/f, \epsilon'_B = f\epsilon_B, \epsilon'_e = f\epsilon_e, f < 1\}$ (and $\theta'_j = \theta_j, p' = p$) is similar to that obtained for the parameter choice $\{E, n_0, \epsilon_B, \epsilon_e\}$ and $f = 1$, for any f in the range $m_e/m_p \leq f \leq 1$.

Let us first consider the velocity fields of the two flows. The flow pattern $\mathbf{v}(\mathbf{r}, t)$ in the modified $f < 1$ flow is similar to that of the $f = 1$ flow, since the energy and density have both been increased by the same factor $1/f$, leaving the ratio E/n unchanged (and since $\theta'_j = \theta_j$). Next, we note that the magnetic field distributions in the two flows are similar. As explained in § 2.1, the energy density in the modified flow is larger than that in the original flow by a factor $1/f$, $e'(\mathbf{r}, t) = e(\mathbf{r}, t)/f$. Decreasing the magnetic field equipartition fraction by a factor f , $\epsilon'_B = f\epsilon_B$, ensures that the magnetic field energy density is similar in both flows.

Finally, we argue that the density and energy distribution of accelerated electrons is the same in both flows. The number density of accelerated electrons is identical in the two flows: the number density of electrons is larger in the modified flow by a factor $1/f$ compared to that in the original flow, $n'_0 = n_0/f$, but only a fraction f of the electrons in the modified flow are accelerated. The total energy density in electrons is also the same in both flows, since $\epsilon'_e e'(\mathbf{r}, t) = \epsilon_e e(\mathbf{r}, t)$. The fact that the electron energy density and accelerated electron density are similar in the two flows does not ensure that the energy distributions of accelerated electrons are similar, since in the modified flow some part of the electron energy density is carried by the non–shock-accelerated electrons. This part is small, however, and therefore the energy distributions of the accelerated electrons are similar in both flows, as long as $m_e/m_p < f$.

To see this, we note that during the relativistic phase of expansion, the characteristic Lorentz factor γ_{e0} of accelerated electrons, $\epsilon_{e0} = \gamma_{e0} m_e c^2$, is approximately determined by the relation $(1-f)\gamma m_e c^2 + f\gamma_{e0} m_e c^2 = \epsilon'_e \gamma m_p c^2 = f\epsilon_e \gamma m_p c^2$. Since afterglow observations imply $\epsilon_e \sim 1$ for $f = 1$, as long as $m_e/m_p < f$ we have $\gamma_{e0} \approx \epsilon_e \gamma m_p / m_e$ independent of f . During the subrelativistic

regime, γ_{e0} is determined by the relation $(1-f)m_e v^2/2 + f\gamma_{e0} m_e c^2 = \epsilon'_e m_p v^2/2 = f\epsilon_e m_p v^2/2$. Here too, γ_{e0} is independent of f , approximately given by $\gamma_{e0} m_e c^2 \approx \epsilon_e m_p v^2/2$, as long as $m_e/m_p < f$. It is important to note here that since we have several examples for which afterglow observations cover both relativistic and subrelativistic evolution phases (e.g., Frail et al. 2000; Berger et al. 2004), the efficiency f should be similar at both stages. This independence of f from γ is not necessarily surprising, since f may be, e.g., a function of m_e/m_p alone.

Since the two flows have similar velocity fields, magnetic field energy distributions, and accelerated electron distributions, the afterglow radiation emitted by the accelerated electrons is similar for the two flows. The presence of a non–shock-accelerated electron population may, however, modify the radiation pattern. This issue is discussed in § 3.

3. SIGNATURES OF LOW EFFICIENCY

Consider the possible presence of a large number of “thermal” electrons that entered the shock at energy $\gamma m_e c^2$ as measured in the shock frame. Assume that they are heated somewhat to a typical energy of $\eta m_e c^2$, where $\eta \ll m_p/m_e$. Here η is a parameter that expresses our ignorance of the plasma physics that governs the electron heating beyond the energy $\gamma m_e c^2$, which the electrons bring into the shock from upstream. The presence of a large population of these thermal electrons at energy $\gamma \eta m_e c^2 \ll \gamma m_p c^2$ (or at $\sim \eta m_e v^2 \ll m_p v^2$) may affect the emitted radiation by producing either a new component of emission or a large synchrotron self-absorption optical depth, thus suppressing the emission from accelerated electrons. Since the energy distribution of the non–shock-accelerated electrons does not extend (by definition) to energies $\gg \gamma \eta m_e c^2$ (or $\gg \eta m_e v^2$), they may affect the emitted radiation at frequencies $\nu \lesssim \tilde{\nu}_m$, where $\tilde{\nu}_m$ is the characteristic synchrotron emission frequency of electrons of energy $\eta \gamma m_e c^2$ (or $\eta m_e v^2$). This frequency is lower by a factor $(\eta m_e / m_p)^2$ than the characteristic synchrotron emission frequency ν_m of accelerated electrons at energy $\gamma m_p c^2$ (or $m_p v^2$). Since the time dependence of the characteristic synchrotron emission frequency of accelerated electrons typically behaves as $\nu_m \sim 10^{18} (t/100 \text{ s})^{-3/2}$, i.e., peaks at X-rays on minute timescale and drops below the optical on a timescale of 10 hr, the characteristic synchrotron frequency of the non–shock-accelerated electrons drops from $\sim 300 \eta^2$ GHz on minute timescale to $\sim 0.3 \eta^2$ GHz on 3 hr timescale,

$$\tilde{\nu}_m \approx 1 \eta^2 \left(\frac{t}{1 \text{ hr}} \right)^{-3/2} \text{ GHz.} \quad (2)$$

This implies that the existence of a large population of non–shock-accelerated electrons can be constrained only through early radio observations, at $\lesssim \eta^{4/3}$ hr delay. Assuming a spread of several times the thermal energy, so that $\eta \gtrsim 3$, the emission and/or absorption of the thermal electrons could be caught with radio follow-up observations within several hours of the GRB. With sufficient preparation, radio follow-up observations may be carried out on minute timescale (D. Frail & G. Taylor 2004, private communication).

The specific emissivity \tilde{j}_m of the non–shock-accelerated electrons at $\tilde{\nu}_m$ is larger than that of the accelerated electrons at ν_m by a factor $1/f$ (since their number is larger by this factor). Thus, if the optical depth at $\tilde{\nu}_m$ is small, the radio intensity produced by the non–shock-accelerated electrons would be larger by a factor $1/f \gg 1$ than the ~ 1 mJy peak intensity characteristic of the

accelerated electrons for cosmological GRBs. The synchrotron self-absorption optical depth at the peak frequency can be estimated using Kirchoff's law, $\tilde{\tau}_m \propto \tilde{j}_m / \tilde{\nu}_m^2 \tilde{T}$, where the effective temperature is $\tilde{T} = \eta \gamma m_e c^2$. From this relation, we find that the ratio of $\tilde{\tau}_m$ to the optical depth τ_m at ν_m is (for small f) $\tilde{\tau}_m / \tau_m \approx (m_p / \eta m_e)^{5/3} f^{-1}$. For the population of accelerated electrons, $\tau_m = (\nu_a / \nu_m)^{5/3}$, where the self-absorption frequency $\nu_a \sim 1$ GHz and independent of time for expansion into uniform medium (e.g., Waxman 1997b). Combining these relations we have, for expansion into uniform medium and small f ,

$$\tilde{\tau}_m \approx \left(\frac{m_p}{m_e} \right)^{5/3} \eta^{-5} f^{-1} n_0 \left(\frac{t}{1 \text{ hr}} \right)^{5/2}. \quad (3)$$

We have here kept the dependence on the ambient medium number density, $n = 10^0 n_0 \text{ cm}^{-3}$, mainly in order to allow a simple generalization to the case of expansion into a nonuniform medium. For expansion into a wind, $n \propto t^{-1}$, and for typical wind parameters $n_0 \approx 1(t/1 \text{ day})^{-1}$ (Livio & Waxman 2000). All the results given here can thus be applied to the wind case by using $n_0 = 1(t/1 \text{ day})^{-1}$ (note that eq. [2] is valid for any density, i.e., has no dependence on n).

The optical depth at $\tilde{\tau}_m$ is larger than unity for $t > t_a$, where

$$t_a \approx 10^{-2} n_0^{-2/5} \eta^2 f^{2/5} \text{ hr}. \quad (4)$$

At $t < t_a$, the self-absorption frequency $\tilde{\nu}_a$, where the optical depth due to the thermal electrons is unity [$\tilde{\tau}_m(\tilde{\nu}_m/\tilde{\nu}_a)^{5/3} = 1$], is

$$\tilde{\nu}_a \approx \frac{m_p}{m_e} n_0^{3/5} \eta^{-1} f^{-3/5} \text{ GHz}. \quad (5)$$

Finally, the specific intensity at $\tilde{\nu}_m$ is given by

$$\begin{aligned} \tilde{f}_m &\approx f^{-1} f_m \min(1, 1/\tilde{\tau}_m) \\ &= f^{-1} \frac{n_0^{1/2} f_m}{1 \text{ mJy}} \min\left[1, (t/t_a)^{-5/2}\right] \text{ mJy}. \end{aligned} \quad (6)$$

Here, $f_m \sim 1$ mJy is the peak intensity characteristic of the accelerated electrons for cosmological GRBs.

The presence of a significant number of non-shock-accelerated electrons ($f \ll 1$) is therefore expected to lead to a large self-absorption optical depth at frequencies $\nu \leq \tilde{\nu}_m$, strongly suppressing the radio flux at these frequencies at early time. A sharp rise in the flux at frequency ν is expected at $t \approx \eta^{4/3} (\nu/1 \text{ GHz})^{-2/3} \text{ hr}$, as $\tilde{\nu}_m$ drops below ν . The ratio of fluxes obtained in the presence and in the absence of a nonaccelerated electron population, \tilde{f}_ν / f_ν , is given for frequencies lower than the self-absorption frequency of the non-shock-accelerated electrons, $\nu \leq \min(\tilde{\nu}_m, \tilde{\nu}_a)$, by the following argument. At frequencies where the optical depth is large, the intensity is proportional to $\nu^2 T$, where T is the effective electron temperature. For the nonaccelerated electrons, $\tilde{T} = \eta \gamma m_e c^2$, and in the absence of electron cooling, $T = \gamma_{e0} m_e c^2$, so that $\tilde{T}/T \approx \eta m_e / m_p$. At early times, the cooling time of accelerated electrons is short compared to the dynamical (expansion) time, and these electrons lose energy and accumulate at lower Lorentz factor, γ_c , where the cooling time is comparable to the dynamical time. At this energy these electrons radiate synchro-

tron photons at frequency $\nu_c = (\gamma_c / \gamma_{e0})^2 \nu_m$, and for typical model parameters

$$\frac{\nu_m}{\nu_c} \approx 10 n_0 \left(\frac{t}{1 \text{ hr}} \right)^{-1}. \quad (7)$$

Thus, for $\nu \leq \min(\tilde{\nu}_m, \tilde{\nu}_a)$ the flux suppression factor is given by

$$\frac{\tilde{f}_\nu}{f_\nu} = \frac{\eta m_e}{m_p} \max\left[1, \left(\frac{\nu_m}{\nu_c}\right)^{1/2}\right] \max\left[1, \left(\frac{\nu}{\nu_a}\right)^{5/3}\right]. \quad (8)$$

The last term on the right-hand side accounts for the fact that $f_\nu \propto \nu^{1/3}$ (rather than $f_\nu \propto \nu^2$) for $\nu > \nu_a$. Cooling of electrons increases ν_a by a factor $\gamma_{e0} / \gamma_c = (\nu_m / \nu_c)^{1/2}$ compared to the case in which cooling is unimportant, which implies

$$\nu_a \approx n_0^{3/5} \max\left[1, 3 n_0^{1/2} \left(\frac{t}{1 \text{ hr}}\right)^{-1/2}\right] \text{ GHz}. \quad (9)$$

The presence of a large number of non-shock-accelerated electrons can be detected through their radio emission only if this emission takes place in an optically thin regime, i.e., at frequencies $\tilde{\nu}_a < \nu < \tilde{\nu}_m$ for $\tilde{\nu}_a < \tilde{\nu}_m$. Examining equations (2) and (5), we find that $\tilde{\nu}_a < \tilde{\nu}_m$ is possible only for $\eta \gg 1$ and moderate f^{-1} . For $\eta = 10$ and $f = 10^{-1}$, for example, the flux at $\tilde{\nu}_m \approx 400(\eta/10)^2 (t/0.4 \text{ hr})^{-3/2} \text{ GHz}$ is $\sim 10(10f)^{-1} \text{ mJy}$ up to $t \approx 0.4(\eta/10)^2 (10f)^{2/5} \text{ hr}$. For $\tilde{\nu}_a < \nu < \tilde{\nu}_m$ and $\nu > \nu_a$ we have

$$\frac{\tilde{f}_\nu}{f_\nu} = \left(\frac{\eta m_e}{m_p} \right)^{-2/3} f^{-1} \min\left[1, \left(\frac{\nu_m}{\nu_c}\right)^{-1/3}\right]. \quad (10)$$

Finally, it is useful to give an estimate of the amplitude of the radio flux typically expected on the relevant timescale. For $\nu > \nu_a$, the flux expected (in the absence of non-shock-accelerated electrons) is

$$\begin{aligned} f_\nu &\approx \max\left[1, \left(\frac{\nu_m}{\nu_c}\right)^{-1/3}\right] \left(\frac{\nu}{\nu_c}\right)^{1/3} f_m \\ &\approx 30 n_0^{5/6} \frac{f_m}{1 \text{ mJy}} \left(\frac{\nu}{10 \text{ GHz}}\right)^{1/3} \left(\frac{t}{1 \text{ hr}}\right)^{1/6} \\ &\quad \times \max\left[1, \left(\frac{\nu_m}{\nu_c}\right)^{-1/3}\right]. \end{aligned} \quad (11)$$

4. DISCUSSION

We have shown that current afterglow observations do not allow one to determine the efficiency of electron acceleration in GRB shocks, i.e., to determine the fraction f of electrons that are “injected” to participate in the process of shock acceleration. While afterglow observations imply that some fraction f of the electron population is accelerated to a characteristic energy ε_{e0} comparable to the postshock proton temperature, $\varepsilon_{e0} \approx \gamma m_p c^2$ for relativistic shocks of Lorentz factor γ or $\varepsilon_{e0} \approx m_p v^2/2$ for nonrelativistic shocks of velocity v , a large fraction, $1 - f \sim 1$, of the electron population may be “left behind” at low energy comparable to the kinetic energy of the electrons propagating to the shock, $\gamma m_e c^2$ for relativistic shocks or $m_e v^2/2$ for nonrelativistic shocks. The resulting electron energy distribution is qualitatively described in Figure 1. Currently testable afterglow

predictions of a model with parameter choice $\{E' = E/f, n'_0 = n_0/f, \epsilon'_B = f\epsilon_B, \epsilon'_e = f\epsilon_e, f < 1\}$ are similar to those obtained for the parameter choice $\{E, n_0, \epsilon_B, \epsilon_e\}$ and $f = 1$, for any f in the range $m_e/m_p \leq f \leq 1$. This implies an uncertainty of factor m_e/m_p in the determination of model parameters. Afterglow observations do not constrain, for example, the values of E and ϵ_e , but rather the values of fE and ϵ_e/f . Note that the value of ϵ_{e0} is independent of f (and $\approx \gamma m_p c^2$ or $\approx m_p v^2/2$).

The existence of non-shock-accelerated electrons will strongly affect the predicted radio emission on short, $\lesssim 1\eta^{4/3}$ hr, timescale. Here, $\eta\gamma m_e c^2$ (or $\eta m_e v^2/2$) is the characteristic energy of non-shock-accelerated electrons ($\eta \ll m_p/m_e$). For $f \ll 1$, a large self-absorption optical depth at $\tilde{\nu}_m \approx 1\eta^2(t/1 \text{ hr})^{-3/2}$ GHz (eq. [3]) would lead to strong suppression of the radio flux at lower frequencies (eq. [8]). As $\tilde{\nu}_m$ drops below an observed frequency ν , at $t \approx \eta^{4/3}(\nu/1 \text{ GHz})^{-2/3}$ hr, the optical depth at this frequency drops below unity, and a sharp brightening is expected. For $\eta \gg 1$ and moderate f^{-1} , the existence of a large population of non-shock-accelerated electrons may be identified through their radio emission (see eq. [10] and the discussion preceding it). For $\eta \gg 1$, the modification of radio emission due to the presence of non-shock-accelerated electrons will persist over timescales significantly larger than 1 hr. The radio signature of these thermal electrons could test for their presence at levels that are energetically insignificant by a large margin (even $f^{-1} \lesssim 1$) and therefore otherwise inconspicuous.

It should be pointed out that afterglow observations already provide interesting constraints on the efficiency of electron acceleration. First, they require similar efficiency f for both relativistic and subrelativistic shocks, since several examples exist of afterglow observations covering both relativistic and subrelativistic evolution phases (e.g., Frail et al. 2000; Berger et al. 2004). This independence of f from γ is not necessarily surprising, since f may be, e.g., a function of m_e/m_p alone. Second, afterglow observations imply that the energy of accelerated electrons is increased to a characteristic energy similar to the proton postshock temperature (with power-law extension to high energies). Finally, the value of f is limited to $f > m_e/m_p$. Early, $\lesssim 1$ hr, radio observations will provide more stringent constraints on the efficiency f (and will hence remove the degeneracy in determining GRB model parameters). As mentioned in § 2, radio spectra can also be used to constrain the energy distribution of accelerated electrons at energies below the characteristic acceleration energy, $\gamma m_p c^2$ (Waxman 1997b), providing further constraints on the acceleration process.

The total, beaming-corrected, energy released in cosmological long-duration GRB explosions, assuming $f = 1$, is typically inferred to be $E_T \sim 10^{51.5}$ ergs (Frail et al. 2000, 2001; Freedman & Waxman 2001; Berger et al. 2003, 2004), with a spread in estimated values of roughly 1 order of magnitude. Since afterglow observations do not constrain E_T but rather fE_T , the true explosion energies are $E_T \sim f^{-1}10^{51.5}$ ergs. For $f \ll 1$, explosion energies $\gg 10^{51.5}$ ergs would naively be inferred for many GRBs. The association of (at least some) GRBs with supernovae (Galama et al. 1998b; Stanek et al. 2003; Hjorth et al. 2003; Bloom 2005) suggests that the total energy is probably not much more than $10^{51.5}$ ergs, ruling out values of $f \ll 1$. Using this argument to infer a conservative lower limit on f , the uncertainties in determining fE_T from afterglow observations should be considered. Uncertainties in determining E may arise from uncertainties in the determination of the observables $\{\nu_m, F_m, \nu_{\text{cool}}, \nu_{\text{sa}}\}$. The main uncertainty here is due to uncertainty in the determination of the self-absorption frequency, which is not known in many cases and determined in

the best cases to within a factor of ~ 2 , leading to uncertainty in E of a similar magnitude (since $E \propto \nu_{\text{sa}}^{-5/6}$; e.g., Wijers & Galama 1999; note that the uncertainty in $n \propto \nu_{\text{sa}}^{25/6}$ and $\epsilon_B \propto \nu_{\text{sa}}^{-5/2}$ is much larger). Moreover, it should be realized that afterglow models are highly idealized (e.g., assuming simple geometry), and various effects that are not taken into account (e.g., acceleration of preshocked plasma by a cosmic-ray precursor) may lead to systematic errors in estimates of model parameters. Thus, estimates of the energy should be considered as order of magnitude estimates. Finally, the energy provided by the supernova to the GRB jet could be higher than that provided to the supernovae ejecta, whose energy is limited by neutrino cooling. Altogether, although $f \sim 1$ is suggested by the energy derived from afterglow observations, $f \geq 1/30$ should be considered a plausible conservative lower limit.

Is there an a priori reason to suspect that f should be small? In the case of plerionic nebulae, such as the Crab, typical shock-accelerated spectra ($p \sim 2.2$) occur above Lorentz factors of order $\gamma_0 \sim 10^4$, and below that the spectra are much flatter, $1.3 \geq p \geq 1.1$ (Weiler & Panagia 1978). Curiously, the low-energy end of these spectra goes well below $\gamma m_e c^2$ (where the bulk Lorentz factor of the preshock wind can be estimated if the total number of electrons and the total energy that have been deposited into the nebulae by the wind are known). These low-energy electrons do not increase the total energy requirements but clearly comprise most of the electrons by number. This raises the question of how most of the electrons in the nebula can have less energy than they had flowing into the shock, and strongly suggests some sort of shock mediation mechanism that redistributes their energy in the form of a hard power law.

Suppose the same sort of low-energy spectra were obtained below $\gamma m_p c^2$ in GRB blast waves. We can express these low-energy electron populations as $f^{-1}(\eta) \simeq (\eta m_e/m_p)^{-p+1}$. Basically, $f^{-1}(\eta)$ is the number of low-energy electrons at energy ηm_e relative to those at m_p . If plerion-like low-energy spectra were obtained in GRB post-afterglow-shock plasmas, they would give values of f of more than $1/30$ at $\eta \geq 1$ but would violate the constraint $d \ln n/d \ln \epsilon \geq 0$ that seems to exist at least for some GRB afterglows. These current limits extend from $1/30 \leq \eta m_e/m_p \leq 1$. Exploring the region $\eta m_e/m_p \leq 1/30$ at $1\nu_{\text{GHz}}$ GHz will, by equation (2), require radio follow-up observations within $(10^{3.5}/\nu_{\text{GHz}})^{2/3}$ hr. The signature of a thermal population of electrons would be (1) a radio “blackout” due to the low brightness temperature of the thermal electrons, followed by (2) a prebrightening, as the emitting area increases, followed by (3) a steep decline, as the emitting frequency of the thermal electrons passes below the observed frequency. All these stages precede the expected rise associated with the eventual passage of ν_m through the observing frequency.

In summary, early observations of emission and self-absorption in GRB afterglows can provide a diagnostic of the low-energy electron spectra in GRB afterglow shocks. This might either change our understanding of them or, at least, close some important loopholes in afterglow theory.

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